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HABIT FORMATION, DYNASTIC ALTRUISM, AND POPULATION DYNAMICS

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We study the general equilibrium properties of two growth models with overlapping generations, habit formation, and endogenous fertility. In the neoclassical model, habits modify the economy's growth rate and generate transitional dynamics in fertility; stationary income per capita is associated with either increasing or decreasing population and output, depending on the strength of habits. In the AK specification, growing population and increasing consumption per capita require that the habit coefficient lie within definite boundaries; outside the critical interval, positive growth is associated with either declining consumption due to overcrowding, or extinction paths with declining population. In both frameworks, habits reduce fertility: the trade-off between second-period consumption and spending for bequests prompts agents to decrease fertility in order to make parental altruism less costly. This mechanism suggests that status-dependent preferences may explain part of the decline in fertility rates observed in most developed economies.

Keywords: Economic Growth, Endogenous Fertility, Habit Formation, Intergenerational Altruism

1. INTRODUCTION

One of the major stylized facts that characterized the development process of industrialized economies is the decline in fertility rates. In developed countries, the transition from rapid population growth to low net fertility rates began with the second phase of industrialization in the 19th century. Birth rates declined faster than mortality rates, yielding a substantial reduction in net population growth—a phenomenon labeled as the *demographic transition*. After World War II, net fertility rates reached exceptionally low levels and fell short of the "replacement threshold" even in countries where fertility had traditionally been high—e.g.,

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Spain and Italy.[1] In less-developed countries, the fertility transition started in the mid-1960s, and it was particularly rapid in East Asia.

In spite of institutional differences, demographic transitions regularly characterized economic development throughout the industrialized world. The study of the causal relations behind the fertility decline attracted the attention of several economists, and the renewed interest in formal growth theories inspired a new body of literature that analyzes endogenous population dynamics. Various explanations for the demographic transition have been advanced. First, declining fertility rates may be due to technological progress that—through its impact on the demand for human capital—reverses the relationship between income and population growth with respect to the regime of Malthusian stagnation [Galor and Weil (2000)]. Second, increasing real wages raise the opportunity cost of having children, and lower fertility generates positive feedback effects on economic growth through capital accumulation [Galor and Weil (1996)]. Third, there may be trade-offs between the quality and quantity of children in parents' desires, that introduce a bias against quantity due to parents' aspirations [Mulligan (1997)], the forces of natural selection [Galor and Moav (2002)], increased longevity [Ehrlich and Lui (1991)], or interactions between education choices and unobservable skills of children [Becker (1991)]. Fourth, fertility decline may be associated with changing patterns in intergenerational transfers—the so-called Caldwell hypothesis [Caldwell (1982)]. At low levels of economic development, the average family size is large as transfers flow from the young to the old. In developed economies, family size is small as the net transfer flow is from parents to children. The idea that population dynamics are governed by the direction of transfers has been formalized in a recent contribution by Blackburn and Cipriani (2005), who assume that agents optimally choose the amount of transfers to both parents and children and show that two-sided altruism generates development paths that are consistent with the Caldwell hypothesis.

Empirical evidence and quantitative analyses suggest that each of these views has its merits, though a monocausal explanation for (i) the low fertility rates currently observed in developed countries and (ii) the demographic transition experienced by most Western economies is unlikely to hold [see Mateos-Planas (2002); Doepke (2005); Lagerlöf (2006)].[2] The aim of this paper is not to challenge previous explanations but rather to investigate an additional mechanism through which fertility choices might have been affected by economic development. The basic idea is that fertility choices interact with status-dependent preferences. In particular, we argue that *habit formation* generates reallocation effects that help explain the decline in fertility rates.

Most if not all theoretical models of fertility assume that intertemporal choices are based on standard time-separable preferences defined over absolute consumption levels. However, there is now a large consensus on the fact that preferences are status- and time-dependent in reality. A growing body of empirical evidence shows that economic agents form habits and tend to assess present satisfaction on the basis of deviations from the standards of living enjoyed in the past [Osborn

(1988); Fuhrer and Klein (1998); Fuhrer (2000); Guariglia and Rossi (2002)]. At the theoretical level, a recent strand of literature investigates the effects of habits on economic growth in general equilibrium models. In this framework, habit formation—also labeled as "internal habits" or "inward-looking preferences"—affects capital accumulation because agents make their savings decisions by comparing current consumption with a psychological benchmark, represented by a weighted average of own past consumption levels [Caroll, Overland, and Weil (1997); Alvarez-Cuadrado, Monteiro, and Turnovsky (2004)]. Quantitative applications suggest that status-dependent preferences may explain various stylized facts—e.g., the hump-shaped time paths that characterized the behavior of saving rates in Japan [Caroll (2000)] and Western Europe in the postwar period [Alvarez-Cuadrado (2008)].

Recent contributions relax the traditional assumption of infinitely lived agents and analyze the consequences of benchmark preferences in overlapping generations models in which consumers optimize over finite horizons. Abel (2005) studies the effects of habits and social status on the allocation of consumption across generations when agents are selfish. Alonso-Carrera, Caballè, and Raurich (2007) extend the model of dynastic altruism of de la Croix and Michel (1999) to include habit formation and show that habits reduce the willingness of individuals to leave bequests under exogenous population growth. This result hinges on a general mechanism of reallocation in life-cycle resources that will be relevant to our conclusions. In particular, we will formalize the idea that persistently low fertility rates can be induced by status-dependent preferences because agents internalize previous standards of living into their saving motives and fertility choices. This reasoning may be linked to the observation that low-reproduction strategies—i.e., intentional choices aimed at preserving the family property and status—ceased to be an exclusive feature of the aristocracy already in the 19th century, as the same behavior gradually spread across the bourgeoisie, landowners, and other social classes [Johansson (1987); Haines (1992)].

At the formal level, our analysis may be interpreted as an extended dynastic model that includes both habit formation and endogenous fertility. In Section 2, we study individual choices in an overlapping generations (OLG) economy, in which the number of children provides utility in the first period of life and second-period utility depends on the gap between current and previous consumption levels. Agents are altruistic toward their descendants, and the opportunity cost of fertility is determined by the amount of bequests that parents give to their children. However, due to habit formation, consumption choices are biased in favor of second-period consumption, and agents tend to reduce the share of resources devoted to bequests. The consequence of this consumption bias is a reduction in fertility rates: being able to choose the number of children, agents reduce fertility in order to make parental altruism less costly. We investigate the implications for economic growth and population dynamics under two technology specifications that are standard in growth theory. Section 3 considers an AK model in which the growth rate of aggregate output is independent of habits. In this framework,

growing population and increasing consumption per capita require that the habit coefficient lie within definite boundaries. Outside the critical interval, positive growth is associated with either declining consumption due to overcrowding or extinction paths with declining population. Section 4 assumes a neoclassical technology with constant returns to scale. Differently from the Ramsey model with habit formation and exogenous population growth [Ryder and Heal (1973)], habits modify the economy's growth rate. Whereas the long-run equilibrium features constant output per capita, the growth rate of aggregate output is determined by the equilibrium rate of population growth—which is endogenous and modified by habit formation. In the long run, stationary income per capita is associated with either increasing or decreasing population and output, depending on the strength of habits. We also simulate the transitional dynamics, showing that a declining transitional path in fertility rates arises and is exclusively due to habit formation. Section 5 discusses the connections between our results and previous literature, and Section 6 concludes.

2. CONSUMPTION AND FERTILITY CHOICES

In each period, indexed by t, total population N_t consists of N_t^o old agents and N_t^y young agents. Young agents supply one unit of work time to firms and save, whereas old agents earn capital income only as a result of previous savings. At the end of period t, each young agent generates n_t children who will become productive workers in the subsequent period: $N_{t+1}^y = N_t^y n_t$. Total population at time $t+1$ thus equals $N_{t+1} = N_t^y(1+n_t)$, and the gross rate of population growth is defined as

$$\frac{N_{t+1}}{N_t} = n_{t-1}\left(\frac{1+n_t}{1+n_{t-1}}\right). \tag{1}$$

As regards individual budget constraints, we allow for the presence of intergenerational transfers in the form of inter-vivos gifts, denoted by $b_t \geq 0$ and defined in the father-to-son direction: if bequest motives are operative, each young agent in period t receives b_t units of output and, in turn, will transfer $b_{t+1}n_t$ units of output to his successors. The individual budget constraints thus read

$$c_t = w_t + b_t - s_t, \tag{2}$$

$$e_{t+1} = r_{t+1}s_t - b_{t+1}n_t, \tag{3}$$

where c is consumption when young, e is consumption when old, w is the wage rate, and r is gross interest on previous savings received during retirement. In general, positive transfers ($b_t > 0$) will arise whenever the degree of dynastic altruism is sufficiently strong to make the bequest motive operative [Thibault (2000)]—a circumstance that can be addressed after deriving the temporary equilibrium of the economy.[3] If bequests are operative in the equilibrium, we have $b_t > 0$, and the resulting equilibrium path exhibits many basic properties of Ramsey-type equilibria in models with an infinitely lived representative agent, such as dynamic

efficiency and policy neutrality; if bequests are not operative in the equilibrium ($b_t = 0$), expressions (2)–(3) reduce to the standard budget constraints of OLG models with selfish agents à la Diamond (1965).[4] In this regard, our strategy will be to study the characteristics of the equilibrium with $b_t > 0$, and derive ex-post existence conditions for an optimal path with positive bequests [see condition (28) below].

Private welfare for each agent alive in $(t, t+1)$ equals

$$W_t = U(c_t, e_{t+1}, n_t) + \delta W_{t+1},$$

where $U(c_t, e_{t+1}, n_t)$ represents direct utility provided by consumption levels and the number of children and $\delta \in (0, 1)$ is the altruism factor, i.e., the weight put by each agent on the welfare of each of the successors.[5] Imposing the limiting condition $\lim_{j\to\infty} \delta^{j-t} W_j = 0$, the dynastic utility function can be written as

$$W_0 = \sum_{j=0}^{\infty} \delta^j U(c_j, e_{j+1}, n_j). \tag{4}$$

Expression (4) is the typical objective function encountered in dynastic models, where the degree of altruism, δ, is analogous to a discount factor imposed at time zero over future generations' direct utilities. As shown, e.g., by de la Croix and Michel (2002: Chapter 5), the assumption of perfect foresight allows us to reinterpret the sequence of individual optimization problems as a single infinite-horizon problem. To analyze the interactions between habit formation and fertility choices, direct utility will be specified as

$$\begin{aligned} U(c_t, e_{t+1}, n_t) &= \bar{u}(n_t) + u(c_t) + \beta v(e_{t+1}, c_t), \\ u_c > 0, u_{cc} \leq 0, \bar{u}_n > 0, \bar{u}_{nn} \leq 0, \\ v_e > 0, v_{ee} \leq 0, \quad \text{and} \quad v_c < 0. \end{aligned} \tag{5}$$

The first element in (5) is a well-behaved utility function $\bar{u}(n_t)$ where the fertility rate appears as a normal good. Having children provides personal satisfaction, and fertility rates are chosen in order to maximize private benefits. The way in which we model fertility choices is thus standard in terms of preferences, but the cost of raising children is specifically linked to our assumption of dynastic altruism. A ceteris paribus increase in the number of children implies an increased cost in terms of second-period gifts. Each agent will thus balance higher direct utility with reduced consumption possibilities in the second period of life, in compliance with the present-value budget constraint

$$b_{t+1} n_t = r_{t+1} \left[w_t + b_t - \left(c_t + e_{t+1} r_{t+1}^{-1} \right) \right]. \tag{6}$$

As regards consumption preferences in (5), $u(c_t)$ is direct utility from first-period consumption, $\beta > 0$ is the individual time-preference factor, and $v(e_{t+1}, c_t)$ embodies the second crucial assumption of our model, i.e., habit formation. More

precisely, $v\,(e_{t+1}, c_t)$ represents second-period utility from *relative* consumption: for a given benchmark level enjoyed when young, c_t, utility from consumption when old increases with direct consumption ($v_e > 0$, $v_{ee} \leq 0$) but is lower the higher is first-period consumption ($v_c < 0$). In this regard we will follow the standard specification of subtractive habits—see (15) below.

Under the assumption of perfect foresight, the solution to the dynastic problem can be found by maximizing (4) subject to (6), using the sequences of consumption levels and fertility rates as control variables. The associated Lagrangean at time t is

$$\mathcal{L}_t = U\,(c_t, e_{t+1}, n_t) + \delta\lambda_{t+1}\frac{r_{t+1}}{n_t}\left[w_t + b_t - \left(c_t + \frac{e_{t+1}}{r_{t+1}}\right)\right] - \lambda_t b_t, \quad \textbf{(7)}$$

where λ represents the dynamic multiplier attached to the individual budget constraint. The optimality conditions of the consumer's problem read

$$\mathcal{L}_{c_t} = 0 \quad \rightarrow \quad (u_{c_t} + \beta v_{c_t})n_t = \delta\lambda_{t+1}r_{t+1}, \quad \textbf{(8)}$$

$$\mathcal{L}_{n_t} = 0 \quad \rightarrow \quad \bar{u}_{n_t}n_t^2 = \delta\lambda_{t+1}r_{t+1}\left[w_t + b_t - \left(c_t + e_{t+1}r_{t+1}^{-1}\right)\right], \quad \textbf{(9)}$$

$$\mathcal{L}_{e_{t+1}} = 0 \quad \rightarrow \quad \beta v_{e_{t+1}}n_t = \delta\lambda_{t+1}, \quad \textbf{(10)}$$

$$\mathcal{L}_{b_t} = 0 \quad \rightarrow \quad \delta\lambda_{t+1}r_{t+1} = \lambda_t n_t. \quad \textbf{(11)}$$

The crucial conditions linking consumption and fertility choices are thus summarized by

$$\lambda_t = u_{c_t} + \beta v_{c_t} = \beta v_{e_{t+1}}r_{t+1}, \quad \textbf{(12)}$$

$$\bar{u}_{n_t} = \lambda_t b_{t+1}r_{t+1}^{-1}. \quad \textbf{(13)}$$

Expression (12) is the Euler condition for consumption allocation, affected by the presence of habits ($v_{c_t} > 0$). Expression (13) characterizes optimal fertility choices and asserts that the marginal cost of bequests—discounted by the prevailing interest rate—must equal the marginal benefit from having children, $\bar{u}_n$. Notice that, in order to interpret b_t as "bequest," we should impose a nonnegativity constraint $b_t \geq 0$ in each period. For the sake of clarity, we will concentrate on the characteristics of interior solutions without specifying further constraints ex ante. Because the main results are derived while assuming specific functional forms, the nonnegativity of bequests will be addressed by checking, ex post, under what conditions parameters are compatible with positive gifts along the optimal path.[6] Let us assume the following specifications:

$$u\,(c_t) = \log c_t, \quad \textbf{(14)}$$

$$v\,(e_{t+1}, c_t) = \log\,(e_{t+1} - \varepsilon c_t)\,, \qquad \varepsilon > 0, \quad \textbf{(15)}$$

$$\bar{u}\,(n_t) = \gamma n_t^{\sigma}, \qquad 0 < \sigma < 1, \quad \textbf{(16)}$$

where $\gamma > 0$ is a weighting parameter for fertility preferences. Expression (15) specifies habit formation according to the "subtractive form" [Alonso-Carrera et al. (2007)], which postulates a precise willingness to overcome previous consumption levels: the higher is ε the stronger is the role of habits in second-period consumption choices. Expression (16) assumes positive but decreasing marginal utility from the number of children.[7] As shown in the following sections, assumptions (14)–(16) allow us to obtain closed-form solutions in the presence of linear returns to aggregate capital and ensure analytical tractability while studying long-run equilibria under neoclassical technologies. We will later argue (Section 4.2) that logarithmic additivity in $U(c_t, e_{t+1}, n_t)$ may increase the generality of our results by ruling out ad hoc complementarities between consumption and fertility in individual preferences.

From (14)–(16), the optimality condition (12) implies the modified Euler equation

$$e_{t+1} = c_t \left[\beta r_{t+1} + \varepsilon \left(1 + \beta\right)\right], \tag{17}$$

according to which, for a given interest rate, the ratio between second- and first-period consumption is higher the stronger is the degree of habit formation. As may be construed, the bias in favor of second-period consumption generates reallocation effects that modify optimal fertility choices. Studying these interactions, and their consequences for economic development, is the central aim of our analysis.

Because technological specifications matter for the nature of the results, we will consider two central paradigms in growth theory, i.e., neoclassical technologies with constant returns to scale versus constant marginal returns to aggregate capital. For the sake of exposition, we begin by considering a simple AK model of endogenous growth.

3. HABITS, FERTILITY, AND ENDOGENOUS GROWTH

This section analyzes the competitive equilibrium of a decentralized economy under laissez-faire. Consumption and saving choices are characterized by the optimality conditions described in Section 2, whereas the production sector is represented by profit-maximizing firms. To analyze situations with linear returns to aggregate capital, we consider Romer's (1989) specification of learning by doing. There exist J identical firms, indexed by j, producing $\tilde{y}_{(j)}$ units of final good by employing $\tilde{k}_{(j)}$ units of capital and $\ell_{(j)}$ units of labor. Each firm's technology is represented by

$$\tilde{y}_{(j)} = (\tilde{k}_{(j)})^{\mu} \left(h_{(j)} \ell_{(j)}\right)^{1-\mu}, \tag{18}$$

where $h_{(j)}$ represents workers' ability and is taken as given by every agent in the economy. In the competitive equilibrium, factor prices thus equal marginal productivities defined at given ability levels. Because firms are of identical size, they employ identical amounts of inputs and produce the same output level, $\tilde{y}_{(j)} = \tilde{y}$. Aggregate output $Y = J\tilde{y}$ equals $Y = K^{\mu} L^{1-\mu}$, where $K = J\tilde{k}$ is aggregate capital and $L = hJ\ell = hN^{y}$ is aggregate efficient labor (recall that labor is supplied

by the young cohort only). The engine of growth is knowledge accumulation due to learning by doing. Following Romer (1989), workers' knowledge is affected by an aggregated externality: h is positively related to the capital stock per worker, $k_t \equiv K_t/N_t^y$, according to the linear relation

$$h_t = \phi k_t = \phi \left(K_t/N_t^y\right),$$

where the constant $\phi > 0$ represents the intensity of learning by doing. Substituting this relation in the aggregate production function, we obtain $Y_t = AK_t$, where the (gross) marginal *social* return from capital, $A \equiv \phi^{1-\mu}$, is constant over time. The gross marginal *private* return from capital equals the equilibrium interest factor

$$r_t = \mu A < A, \tag{19}$$

which is constant over time. Because the equilibrium wage rate reads

$$w_t = (1-\mu)\left(Y_t/N_t^y\right), \tag{20}$$

we can substitute (19) and $K_{t+1} = N_t^y s_t$ in (2)–(3) to obtain the aggregate resource constraint of the economy,

$$K_{t+1} = AK_t - C_t - E_t, \tag{21}$$

where $C_t \equiv N_t^y c_t$ and $E_t = N_t^o e_t$ represent aggregate consumption of young and old agents, respectively. Exploiting (14)–(16), individual consumption and fertility choices imply the following (all proofs are in the Appendix).

LEMMA 1. *In an interior solution, aggregate consumption of both cohorts grows at the same constant rate*

$$C_{t+1}/C_t = E_{t+1}/E_t = \delta\mu A \tag{22}$$

in each period $t = 0, \ldots, \infty$, *and the share consumed when old is higher the stronger is habit formation*:

$$\frac{C_t}{E_t} = \frac{\delta\mu A}{\mu A\beta + \varepsilon + \varepsilon\beta}. \tag{23}$$

Expression (23) embodies the reallocation effect induced by habit formation. A higher ε corresponds to a stronger willingness to postpone consumption in order to overcome historical standards of living and results in higher shares of output consumed by old agents in each period. Expression (22) is conceptually analogous to the Keynes–Ramsey rule and suggests that a typical balanced growth equilibrium arises in the economy, at least in terms of *aggregate* variables. We use italics in order to stress that positive net growth in aggregate output is not necessarily associated with a continuous rise in individual consumption: equilibrium fertility rates generally differ from output growth rates, so that declining per capita incomes

due to excessive birth rates—or increasing per capita incomes induced by declining population—are not remote possibilities. This point will be addressed immediately after the necessary proof of knife-edge balanced growth:

LEMMA 2. *In an interior solution, the consumption-output share of the young equals*

$$\psi \equiv \frac{C_t}{Y_t} = \frac{(1-\delta\mu)\,\delta\mu A}{\mu A\,(\beta+\delta)+\varepsilon\,(1+\beta)} \tag{24}$$

in each period, and the economy exhibits a balanced growth path with

$$K_{t+1}/K_t = Y_{t+1}/Y_t = \delta\mu A \tag{25}$$

in each period $t = 0, \ldots, \infty$.

An important consequence of knife-edge balanced growth is that individual bequests grow at the same rate as individual consumption. In particular, the effect of habit formation is to reduce equilibrium bequests relative to first-period consumption. To see this formally, define $z_t \equiv b_t/c_t$, and rewrite the lifetime budget constraint (6) as (see Appendix)

$$z_{t+1} = \frac{1}{\delta} z_t + \frac{1}{\delta}\left[\frac{1-\mu}{\psi} - \frac{(\mu A+\varepsilon)\,(1+\beta)}{\mu A}\right]. \tag{26}$$

Because $\delta < 1$, it follows from (26) that, if a steady-state $z_{ss} > 0$ exists, the bequest-consumption ratio jumps at this stationary level at $t = 0$ and is constant thereafter. Imposing stationarity in (26), we obtain

$$z_{ss} = \frac{1}{1-\delta}\left[\frac{(\mu A+\varepsilon)\,(1+\beta)}{\mu A} - \frac{1-\mu}{\psi}\right]. \tag{27}$$

Expression (27) implies that bequests are operative ($z^{ss} > 0$) only if the term in square brackets is positive. From (24), this requires

$$\frac{1-\mu}{(1-\delta\mu)\,\delta} < \frac{\mu A+\varepsilon+\beta\,(\mu A+\varepsilon)}{\mu A\delta+\varepsilon+\beta\,(\mu A+\varepsilon)}. \tag{28}$$

Satisfying the above inequality is a necessary condition to have interior solutions to the dynastic problem. We thus restrict our attention to combinations of parameters that satisfy (28). Recalling that ψ depends on ε, expression (27) implies that an increase in the strength of habits would reduce equilibrium bequests relative to first-period individual consumption: in line with the results of Alonso-Carrera et al. (2007), we have[8]

$$\frac{\partial z_{ss}}{\partial \varepsilon} = \left(\frac{1}{1-\delta}\right)\left(\frac{1+\beta}{\mu A}\right)\left[1 - \frac{1}{(1-\delta\mu)\,\delta}\right] < 0. \tag{29}$$

We now have all the elements to analyze equilibrium fertility rates. The following propositions show that (i) the equilibrium fertility rate is negatively related to the

strength of habits and (ii) positive population growth is compatible with increasing per capita incomes only if the degree of habit formation falls within a precise range of values.

PROPOSITION 1. *Along the equilibrium path, the equilibrium fertility rate is constant: population grows at the equilibrium rate*

$$n_{ss} = \left[z_{ss} \frac{\delta \mu A}{\gamma \sigma \left(\mu A + \varepsilon \right)} \right]^{\frac{1}{\sigma}}, \tag{30}$$

with $\partial n_{ss} / \partial \varepsilon < 0$ *(stronger habits reduce population growth).*

Proposition 1 and expression (29) imply that habit formation has intergenerational impact along two distinct dimensions: stronger habits (i) reduce the share of expenditures devoted to bequests and (ii) reduce population growth. These results have an intuitive interpretation. Habits induce a bias in intertemporal choices that produces a reallocation in favor of second-period consumption. Given the trade-off between second-period consumption and bequests, agents are made more "egotistical" by stronger habits—in the sense that they will reduce fertility rates in order to make parental altruism less costly.

A peculiar feature of this model is that habit formation does not modify the growth rate of aggregate output, whereas it determines fertility rates. This implies that habits modify the dynamics of individual incomes, leaving output growth unaffected at the economy level. In particular, positive growth in aggregate output is not necessarily associated with growing population and increasing consumption per capita. As shown below, this situation arises only if ε lies within definite boundaries.

PROPOSITION 2. *Assume that parameters are compatible with positive growth in aggregate output and positive bequests* ($\delta \mu A > 1$ *and* $z_{ss} > 0$*). There generally exist a couple of critical levels* ε' *and* ε''*, with* $\varepsilon' < \varepsilon''$*, such that* $\varepsilon = \varepsilon'$ *implies* $n_{ss} = \delta \mu A > 1$ *and* $\varepsilon = \varepsilon''$ *implies* $n_{ss} = 1$*. As a consequence, we may have three cases:*

i. *(Extinction path) if* $\varepsilon > \varepsilon''$ *population declines and per capita incomes grow faster than aggregate output;*
ii. *(Overcrowding) if* $\varepsilon < \varepsilon'$ *population grows faster than aggregate output, implying declining per capita incomes;*
iii. *(Non-degenerate growth) if* $\varepsilon' < \varepsilon < \varepsilon''$*, aggregate output, per capita incomes, and population increase over time.*

Proposition 2 directly follows from $\partial n_{ss} / \partial \varepsilon < 0$, and draws a clear-cut distinction between the dynamics of aggregate versus per capita variables. In case (i), habits are very strong and consumption per capita grows faster than aggregate output. The excessive willingness to consume prompts agents to choose low fertility rates, associated with declining population, $n_{ss} < 1$. In the opposite situation—case (ii)—habits are very weak and fertility rates are too high to guarantee sustained

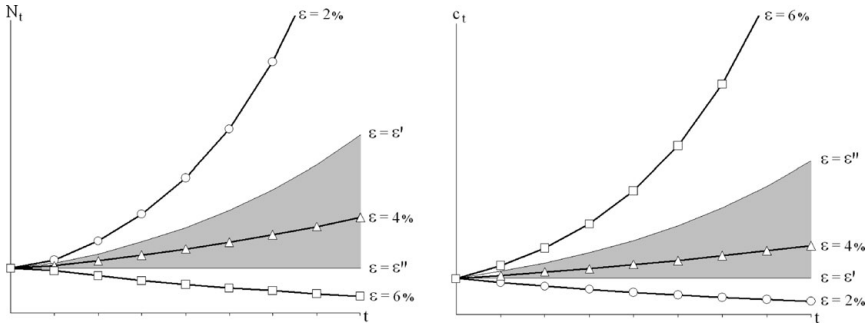

FIGURE 1. Dynamics of population and individual consumption (first period) for different values of ε. Parameter values are $\beta = 0.75$, $\gamma = \delta = \sigma = 0.5$, and $\mu = 0.6$. Nondegenerate growth arises in all paths falling in the gray-shaded areas delimited by the critical values ε' and ε''.

consumption standards for future generations: the gross rate of population growth exceeds $\delta\mu A$, and individual incomes decline over time. The bottom line is that positive growth in aggregate output is associated with a growing population and increasing per capita incomes if and only if the coefficient of habit formation falls within definite boundaries—i.e., the case of "nondegenerate growth" arising when $\varepsilon' < \varepsilon < \varepsilon''$. Numerical substitutions suggest that fertility rates are quite sensitive to the coefficient of habit formation: in the example reported in Figure 1, the values for ε delimiting nondegenerate growth are $\varepsilon' = 3.24\%$ and $\varepsilon'' = 4.89\%$. If the coefficient of habit formation lies outside this interval, overcrowding or extinction paths immediately arise.[9]

The above results are useful in clarifying the negative impact of habits on equilibrium fertility rates. The knife-edge character of the balanced-growth path clearly hinges on the assumption of linear returns, and a fixed interest rate makes the economy's growth rate independent of population growth. Results change in the neoclassical model, in which decreasing marginal returns to capital imply that accumulation rates, and therefore economic growth, are crucially determined by fertility rates. This implies that habit formation modifies the economy's growth rate both in the short and in the long run, as shown below.

4. HABITS, FERTILITY, AND NEOCLASSICAL GROWTH

The neoclassical specification is easily obtained by ruling out learning by doing from the previous model. Assuming that h is a fixed constant in firms' technologies (18), we can define a constant proportionality factor $H = h^{1-\mu}$ and rewrite aggregate output according to the standard Cobb–Douglas form

$$Y_t = H K_t^{\mu} \left(N_t^{y}\right)^{1-\mu}, \tag{31}$$

where K now exhibits decreasing marginal returns. In terms of capital per worker, $k_t \equiv K_t/N_t^y$, the intensive-form technology reads $y_t = f(k_t) = Hk_t^{\mu}$, and equilibrium rates of reward equal

$$r_t = f'(k_t) = \mu H k_t^{\mu-1} \quad \text{and} \quad w_t = (1-\mu) f(k_t). \tag{32}$$

The aggregate constraint of the economy and the first-order conditions imply the following dynamic relations:

$$n_t k_{t+1} = f(k_t) - c_t - (e_t/n_{t-1}), \tag{33}$$

$$e_{t+1} = c_t[\beta f'(k_{t+1}) + \varepsilon(1+\beta)], \tag{34}$$

$$\lambda_t n_t = \lambda_{t+1}\delta f'(k_{t+1}), \tag{35}$$

$$n_{t+1}^{\sigma} = \frac{1}{\delta} n_t^{\sigma} + \frac{1}{\sigma\gamma}\left[(1-\mu) f(k_{t+1}) \lambda_{t+1} - (1+\beta)\right], \tag{36}$$

$$\lambda_t c_t = f'(k_{t+1})\left[f'(k_{t+1}) + \varepsilon\right]^{-1}. \tag{37}$$

Expression (33) is the aggregate constraint of the economy in terms of capital per worker; equations (34)–(35) are the optimality conditions (17) and (11); expressions (36)–(37) are derived in the Appendix and characterize the joint dynamics of shadow prices and fertility rates along the optimal path. We begin our analysis by studying the characteristics of the steady-state equilibrium. Subsequently, we validate the usual interpretation of the steady-state equilibrium as the long-run equilibrium of the economy by means of a numerical simulation that analyzes transitional dynamics (Section 4.2).

4.1. Steady-State analysis

Imposing steady-state conditions in system (33)–(37), and denoting by subscript $*$ stationary values, we obtain

$$n_* k_* = f(k_*) - c_* - (e_*/n_*), \tag{38}$$

$$e_* = c_*[\beta f'(k_*) + \varepsilon(1+\beta)], \tag{39}$$

$$f'(k_*) = n_*/\delta, \tag{40}$$

$$n_*^{\sigma} = \frac{1}{\delta} n_*^{\sigma} + \frac{1}{\sigma\gamma}\left[(1-\mu) f(k_*) \lambda_* - (1+\beta)\right], \tag{41}$$

$$c_* \lambda_* = f'(k_*)\left[f'(k_*) + \varepsilon\right]^{-1}, \tag{42}$$

which is a system of five equations in five unknowns $(n_*, k_*, c_*, e_*, \lambda_*)$. As shown in the Appendix, the equilibrium condition determining the stationary fertility rate can be written as

$$g^a(n_*) = g^b(n_*), \tag{43}$$

where

$$g^a(n_*) = n_*^{\sigma}, \tag{44}$$

$$g^b(n_*) = \frac{\delta(1+\beta)}{\sigma\gamma(1-\delta)} - \frac{(1-\mu)}{\sigma\gamma(1-\delta)(1-\mu\delta)}\left[\frac{n_*(\delta+\beta)+\varepsilon\delta(1+\beta)}{n_*+\delta\varepsilon}\right]. \tag{45}$$

The equilibrium can be characterized by studying the properties of (44) and (45). We will label as "well-defined" the equilibria associated with $n_* > 0$ and as "feasible" only those implying $n_* > \delta$. The reason for the latter restriction is that, from (40), a candidate equilibrium with $n_* < \delta$ would imply $f'(k_*) < 1$—that is, negative net rents from capital. It is instructive to begin with the case of nonexistent habits.

PROPOSITION 3. *If habits are inactive, $\varepsilon = 0$, the steady-state equilibrium is unique.*

Proposition 3 is described in Figure 2, which represents the equilibrium condition (43) in the $g^i - n_*$ plane. When $\varepsilon = 0$, function $g^b(n_*)$ reduces to a horizontal straight line. Because $g^a(n_*)$ is strictly increasing and $g^a(0) = 0$, there can be only one intersection, associated with condition (43). The resulting equilibrium is well-defined and feasible provided that parameters are such that $n_* > \delta$ (see Appendix).

Introducing habit formation, $\varepsilon > 0$, function $g^b(n_*)$ becomes increasing and concave: from (45), we have

$$\frac{\partial g^b(n_*)}{\partial n_*} = \frac{\varepsilon\delta(1-\mu)}{\sigma\gamma(1-\mu\delta)(n_*+\varepsilon\delta)^2} > 0,$$

$$\frac{\partial^2 g^b(n_*)}{\partial n_*^2} = -\frac{2\varepsilon\delta(1-\mu)}{\sigma\gamma(1-\mu\delta)(n_*+\varepsilon\delta)^3} < 0.$$

In this case, the equilibrium can be characterized as follows. Two important properties of $g^b(n_*)$ are that

$$\frac{\partial g^b(n_*)}{\partial \varepsilon} = -\frac{n_*(1-\mu)}{\gamma\sigma(1-\mu\delta)(n_*+\varepsilon\delta)^2} < 0, \tag{46}$$

$$\lim_{n_*\to\infty} g^b(n_*)\Big|_{\varepsilon>0} = \lim_{\varepsilon\to 0} g^b(n_*). \tag{47}$$

Expression (46) implies that a ceteris paribus increase in ε moves $g^b(n_*)$ southwest in the $g^i - n_*$ plane. Expression (47) follows from (45) and asserts that $g^b(n_*)$ is asymptotically horizontal and bounded from above by the value associated with inactive habits, $\lim_{\varepsilon\to 0} g^b(n_*)$. These results imply that, starting from the case of inactive habits, subsequent increases in ε generate downward shifts in

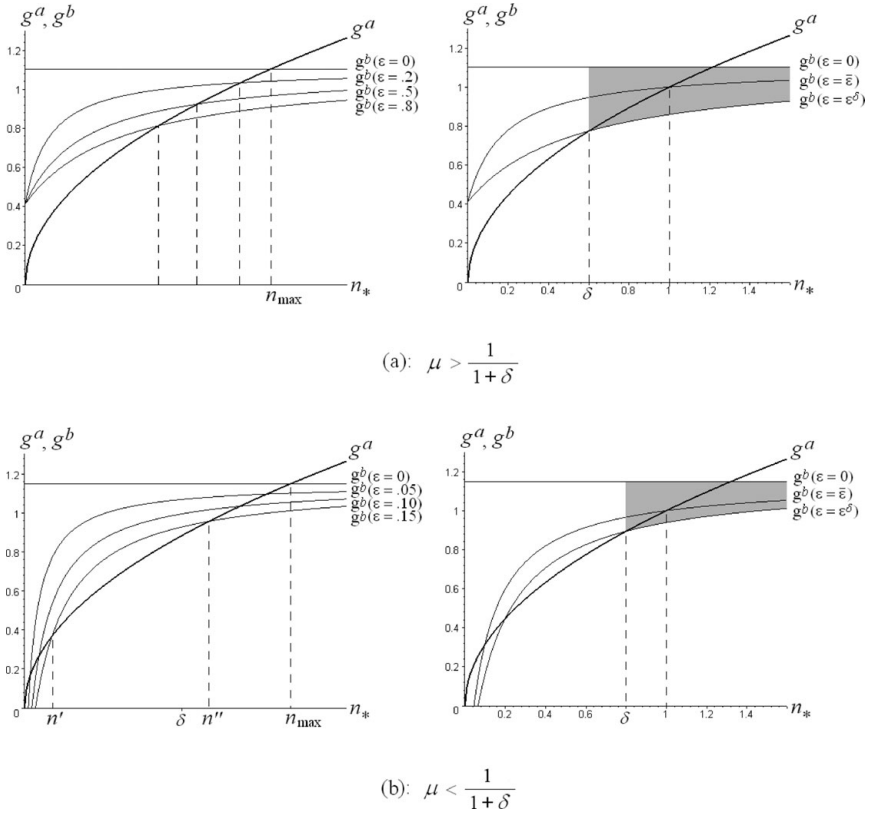

FIGURE 2. Long-run equilibria in the neoclassical model for different values of ε. Diagram (a): unique intersections $g^a = g^b$ and feasible equilibria. Diagram (b): multiple intersections $g^a = g^b$ and unique feasible equilibria.

the $g^b(n_*)$ function for any positive fertility rate. Because $g^a(n_*)$ is increasing and independent of habits, all intersections satisfying the equilibrium condition $g^a = g^b$ will necessarily be associated with *lower* fertility rates with respect to the case $\varepsilon = 0$. This result is described in the left graphs of Figure 2, where the highest equilibrium fertility rate is the habit-free value, denoted as $n_{\max}$.

As both $g^a(n_*)$ and $g^b(n_*)$ are concave, we may have either one or multiple intersections satisfying (43), depending on the constellation of parameters. However, the existence of multiple intersections does not imply multiple equilibria, as shown in the following.

PROPOSITION 4. *For any $\varepsilon < 1$, there may exist only one feasible steady state. This equilibrium is characterized by an intersection in which $g^b(n_*)$ cuts $g^a(n_*)$ from above.*

The intuition behind Proposition 4 is described in graphical terms in Figure 2. As shown in the Appendix, we may have two cases. When $\mu > (1+\delta)^{-1}$, we

may have only one intersection $g^a = g^b$, and this equilibrium is characterized by $g^b(n_*)$ cutting $g^a(n_*)$ from above. This case is reported in Figure 2, diagram (a). When $\mu < (1+\delta)^{-1}$, the vertical intercept of $g^b(n_*)$ is negative, and condition $g^a = g^b$ can be satisfied by two intersections denoted by n' and n'', with $n' < n''$. This situation arises in Figure 2, diagram (b), with $\varepsilon = 0.15$. The "high" intersection n'' has the same properties as the unique equilibrium arising in diagram (a). The "low" intersection n', instead, satisfies the condition $g^a = g^b$ with $g^b(n_*)$ cutting $g^a(n_*)$ from below. However, the low intersection cannot be a feasible equilibrium because the feasibility condition $n' > \delta$ is necessarily violated (see Appendix). As a consequence, a feasible steady state is exclusively characterized by an intersection in which $g^b(n_*)$ cuts $g^a(n_*)$ from above: this is the only possible equilibrium in case (a) and the only feasible equilibrium, n'', in case (b) of Figure 2.

This characterization of feasible equilibria has three main implications. First, a ceteris paribus increase in the strength of habits reduces the optimal fertility rate in any feasible steady state. As shown in the left graphs of Figure 2, g^b shifts downward following an increase in ε, and the equilibrium fertility rate consequently shrinks.

Second, whenever a feasible equilibrium exists without habits, there is always a range of positive values of ε for which feasible equilibria also exist with habit formation. In fact, if $n_{\max} > \delta$, it is always possible to define a critical value of habit formation $\varepsilon^\delta > 0$ such that $\varepsilon = \varepsilon^\delta$ implies an equilibrium with $n_* = \delta$. This critical level can be calculated by imposing $n_* = \delta$ in condition (43) to obtain

$$\varepsilon^\delta = \frac{(1-\mu\delta)[\delta(1+\beta) - \delta^\sigma\sigma\gamma(1-\delta)] - (1-\mu)(\delta+\beta)}{(1-\mu\delta)[\delta^\sigma\sigma\gamma(1-\delta) - \delta(1+\beta)] + (1+\beta)(1-\mu)}. \tag{48}$$

Given Proposition 3 and result (46), any value of ε exceeding ε^δ would generate equilibria that violate feasibility ($n_* < \delta$), whereas any degree of habit formation falling within the interval $\varepsilon \in (0, \varepsilon^\delta)$ yields feasible steady-state equilibria. This is the range of values of ε that is relevant for studying the long-run behavior of the system. In graphical terms, the "relevant region" is represented by the gray-shaded areas in the right graphs of Figure 2.

Third, if parameters allow for positive population growth when habits are inactive, there always exists a critical degree of habit formation associated with constant population, and a subset of feasible equilibria where population declines due to the presence of habits:

LEMMA 3. *Provided that $n_{\max} > 1$, there exists a critical value $\bar{\varepsilon} < \varepsilon_\delta$ such that $\varepsilon = \bar{\varepsilon}$ implies $n_* = 1$. Hence, the set of feasible equilibria with $\varepsilon > 0$ includes two subsets of equilibria respectively associated with $n_* > 1$ and $n_* < 1$.*

The intuition behind Lemma 3 is that, because $\delta < 1$, the special case of constant population $n_* = 1 > \delta$ lies in the interior of the relevant region; see Figure 2. The consequence is that different degrees of habit persistence determine whether long-run population growth rates will be positive or negative. On the one hand,

this result looks similar to that found in the AK model: strong habit persistence may imply declining population. On the other hand, the assumption of decreasing returns to capital yields quite different implications for the economy's growth rate. In the stationary equilibrium of the neoclassical model, consumption and capital per worker are constant, and aggregate output grows at the same rate as population. Formally, because $f'(k_*) = \mu H k_*^{\mu-1}$, condition (40) reads

$$k_* = \left(\frac{\mu\delta H}{n_*}\right)^{\frac{1}{1-\mu}}, \tag{49}$$

and implies that $K_{t+1}/K_t = N_{t+1}/N_t = Y_{t+1}/Y_t = n_*$ in this equilibrium. Hence, *habit formation modifies the economy's growth rate through the fertility rate*. This is an important difference with respect to previous literature on habit formation (see Section 5) and also with respect to the AK model of Section 3. The characteristics of the neoclassical equilibrium are summarized in the following.

PROPOSITION 5. *Provided that* $n_{\max} > 1$, *we may have three cases: (i) if* $0 < \varepsilon < \bar{\varepsilon}$, *the steady-state equilibrium features increasing output and population; (ii) if* $\bar{\varepsilon} < \varepsilon < \varepsilon_\delta$, *the steady-state equilibrium features declining output and population; (iii) if* $\varepsilon = \bar{\varepsilon}$, *the steady-state equilibrium features constant output and population.*

Proposition 5 can be interpreted as follows. Households internalize the effect of habits in their bequest and saving motives, reducing fertility in order to make parental altruism less costly. However, in a neoclassical world, fertility determines the economy's growth rate, which remains strictly positive if and only if habits are relatively weak, $\varepsilon < \bar{\varepsilon}$. Excessive habit formation, $\varepsilon > \bar{\varepsilon}$, induces long-run equilibria in which constant per capita incomes are associated with falling population and declining aggregate output.

4.2. Equilibrium Dynamics

The preceding Section postulates the usual interpretation of the steady-state equilibrium as the long-run equilibrium of the economy. Two questions that still have to be addressed relate to the dynamic stability of the stationary equilibrium and the effects of habit formation on transitional dynamics. The starting point of the analysis is a three-by-three dynamic system that is obtained from suitable substitutions in (33)–(37). As shown in the Appendix, the dynamic behavior of the economy is fully determined by three equations involving the crucial variables k, λ, and n:

$$k_{t+1} = \frac{1}{n_t}\left[f(k_t) - \frac{f'(k_{t+1})}{\lambda_t(f'(k_{t+1}) + \varepsilon)} - \frac{\beta f'(k_{t+1}) + \varepsilon(1+\beta)}{\lambda_t\delta(f'(k_{t+1}) + \varepsilon)}\right], \tag{50}$$

$$\lambda_{t+1} = \frac{\lambda_t n_t}{\delta f'(k_{t+1})}, \tag{51}$$

$$n_{t+1} = \left[\frac{1}{\delta} n_t^{\sigma} + \frac{1}{\sigma\gamma}\left[(1-\mu)\, f\,(k_{t+1})\,\lambda_{t+1} - (1+\beta)\right]\right]^{1/\sigma}. \qquad (52)$$

From (50)–(52), the evolution of capital per capita depends on the expected interest rate, and the dynamics of k_{t+1} are implicitly determined by equation (50) for a given set of parameters $\{\mu, H, \gamma, \sigma, \beta, \delta, \varepsilon\}$.[10] As before, it is instructive to begin with the case of inactive habits. In this case, it is possible to show analytically that $\varepsilon = 0$ implies no transition in fertility rates.

PROPOSITION 6. *When habits are inactive, the fertility rate is constant at each point in time along the optimal path.*

The intuition behind Proposition 6 is provided by the equilibrium condition for consumption allocation (see Appendix),

$$\frac{C_t}{E_t} = \frac{\delta\,(r_t + \varepsilon)\, r_{t+1}}{(r_{t+1} + \varepsilon)\,[\beta r_t + \varepsilon\,(1+\beta)]}. \qquad (53)$$

Expression (53) shows that the shares of output going to young and old agents are generally time-varying, as they depend on capital accumulation. Setting $\varepsilon = 0$, however, expression (53) reduces to $C_t/E_t = \delta/\beta$. Proposition 6 can thus be interpreted as follows: in the presence of habits, the allocation of output among cohorts varies over time because individual choices adjust to changing standards of living during the development process. Because the number of children is determined by utility maximization, this mechanism characterizes both consumption and fertility choices. Without habits, output allocation is time-invariant as no adjustment in consumption, or in fertility rates, is called for by changing standards of living.

In general, the presence of habits yields transitional variations in fertility rates. Given the complexity of the dynamic system (50)–(52), this issue can be addressed only numerically. As a first step, we considered several different constellations of parameters, and derived the respective eigenvalues from the Jacobian matrix. In this regard, numerical results show that system (50)–(52) exhibits saddle-point stability for a wide range of parameter values, with one stable and two unstable eigenvalues. As a second step, we performed a numerical simulation of the full dynamics of the economy along the optimal path, with a special focus on the transitional impact of habits on fertility decisions. To circumvent the disadvantages of methods based on linearization, we used backward iteration to characterize approximate solutions to the Euler equations [Strulik and Brunner (2002); Heer and Maussner (2005); see Appendix for details]. As noted above (note 10), the dynamics of k_{t+1} are implicitly determined by equation (50) for a given set of parameters $\{\mu, H, \gamma, \sigma, \beta, \delta, \varepsilon\}$ and determine the respective entries in the Jacobian matrix of the system by applying the implicit-function theorem. As a benchmark case, we specify the set of parameters as

$$\mu = 0.36,\ H = 10,\ \gamma = 0.12,\ \sigma = 0.5,\ \beta = 0.935,\ \delta = 0.93,\ \varepsilon = 0.015,$$

which implies, in light of (38)–(42), the steady-state values

$$c_* = 6.24008,\ e_* = 6.84885,\ n_* = 1.06281,\ b_* = 0.42048,$$
$$k_* = 6.0066,\ \lambda_* = 0.15817.$$

The Jacobian of (50)–(52) evaluated at the steady-state, $\bar{J}_*$, is equal to

$$J_* = \begin{vmatrix} \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_* & \left.\frac{\partial k_{t+1}}{\partial \lambda_t}\right|_* & \left.\frac{\partial k_{t+1}}{\partial n_t}\right|_* \\ \left.\frac{\partial \lambda_{t+1}}{\partial k_t}\right|_* & \left.\frac{\partial \lambda_{t+1}}{\partial \lambda_t}\right|_* & \left.\frac{\partial \lambda_{t+1}}{\partial n_t}\right|_* \\ \left.\frac{\partial n_{t+1}}{\partial k_t}\right|_* & \left.\frac{\partial n_{t+1}}{\partial \lambda_t}\right|_* & \left.\frac{\partial n_{t+1}}{\partial n_t}\right|_* \end{vmatrix}$$
$$= \begin{vmatrix} 1.075268818 & 76.06568876 & -5.697822964 \\ 0.018122124 & 2.281978849 & 0.0528000960 \\ 11.87479223 & 1259.405296 & 0.5652977844 \end{vmatrix},$$

and the associated eigenvalues read

$$\nu = \begin{vmatrix} 2.39907513941288996 \\ 1.07526893420792736 \\ 0.44820137777951952 \end{vmatrix}.$$

With two unstable eigenvalues ($\nu_1, \nu_2 > 1$) and one real stable eigenvalue ($\nu_3 < 1$), the dynamic system (50)–(52) exhibits saddle-point stability with monotone convergence. The results of the numerical exercises are presented in Figures 3 and 4.

We analyze the effect of habits by considering three scenarios: (i) $\varepsilon = 0$, represented by the solid line; (ii) $\varepsilon = 1\%$, dashed line; and (iii) $\varepsilon = 1.5\%$, dotted line. Looking at Figure 3, it becomes apparent that habits do not play an important role in very early stages of economic development. As the economy becomes richer, parents internalize the existence of habits in their bequest and fertility choices, and the effects of habit formation become relevant. Because agents tend to allocate more resources in favor of second-period consumption, savings increase with the strength of habits, thereby fueling capital accumulation. To reduce the private cost of having children, agents choose lower fertility rates for higher values of the habit coefficient. The bequest-consumption ratio, represented by $z_t = b_t/c_t$, is declining over time when habits are active, and the long-run value is inversely related to the degree of habit persistence. This result confirms the intuition behind the reallocation effects induced by habit formation and is in line with our previous conclusions in the AK model—see expression (29) in Section 3.[11]

Figure 4 embodies two results. First, by virtue of the mechanism mentioned above, fertility rates decline during the transition, i.e., net population growth rates are progressively reduced by economic development. Second, the simulation confirms Proposition 6, by which fertility rates are constant during the transition

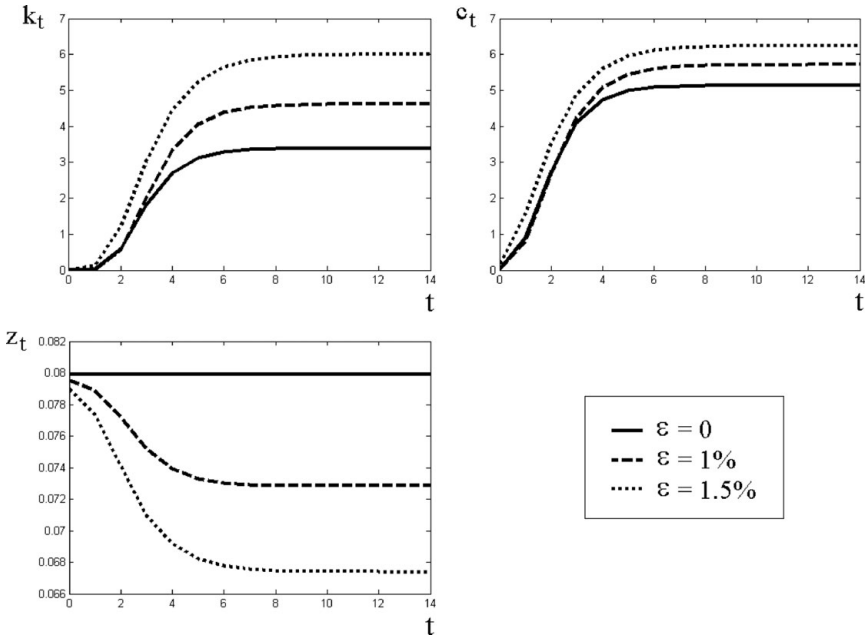

FIGURE 3. Transitional dynamics of capital per capita k_t, first-period consumption c_t, and bequest-consumption ratio $z_t = b_t/c_t$.

when $\varepsilon = 0$. In other words, the "demographic transition" depicted in Figure 4 is exclusively due to habit formation.

It should be stressed that the assumption of logarithmic preferences in consumption is relevant for Proposition 6. If consumption preferences display an elasticity of intertemporal substitution different from unity, the allocation of consumption over the life cycle is affected by interest rates, independently of the presence of habits. In this case, consumption shares are generally time-varying in the short run, and fertility rates likely exhibit transitional dynamics even without habits. Hence, the transitional effects of habits on fertility would interact with those stemming from nonlogarithmic preferences. However, our long-run results should remain valid even with other types of preferences. The reason is that the reallocation effect—i.e., the fact that habits reduce the share of resources devoted to bequests—is a general mechanism that does not hinge on logarithmic forms. As shown by Alonso-Carrera et al. (2007), who assume exogenous population growth, habits make the operativeness of bequests less likely even with generic additive preferences. Building on this mechanism, the peculiar result of our model remains: given the possibility of modifying the fertility rate, agents are able to reduce the total cost of bequests by decreasing the number of children. In this respect, notice that our assumption of logarithmic preferences may increase, rather than limit, the generality of our conclusions. The reason is that, in both the AK and

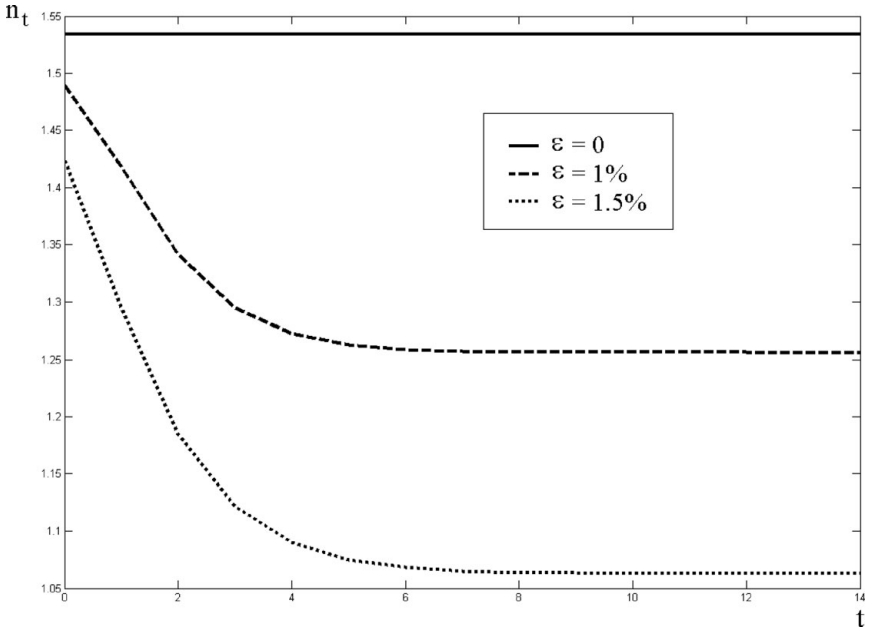

FIGURE 4. Transitional dynamics of fertility rates for increasing strength of habits.

the neoclassical model presented above, the negative long-run impact of habits arises without making ad hoc assumptions of complementarity, or substitutability, between consumption and fertility in individual preferences.

5. CONNECTIONS WITH PREVIOUS LITERATURE

With respect to previous literature on demographic transition, the main distinctive feature of our analysis is given by our main result: habit formation—and, in general, status-dependent preferences—may constitute an important part of the explanations for the decline of fertility rates exhibited by developed economies. To our knowledge, previous studies did not address this issue. The old-age security approach postulates that present fertility choices are driven by the expectation that children will provide support to their parents in the future, so that variations in equilibrium fertility rates are determined by uncertainty over future incomes [Nishimura and Zhang (1995)] and linked to the degree of risk aversion [Sah (1991)]. In the standard framework in which children provide direct utility to their parents, the fertility decline may be due to increased productivity from technological progress [Galor and Weil (2000)], rising real wages that increase the opportunity cost of having children [Galor and Weil (1996)], and possible trade-offs between the quality and quantity of children [Becker (1991)]. In this regard, we may stress that our results do not hinge on interactions between the opportunity cost of fertility and technological development. In the present analysis, the central

element is the *psychological* cost of having children: low fertility rates originate in the distorted perception of "joy from consumption" that old agents have in the second period of life. Our reasoning also differs from explanations based on quality–quantity trade-offs. If parents reduce the number of children in order to provide better education to each successor, the roots of low fertility rates are to be found in parental altruism, whereas our analysis suggests the conclusion—perhaps less pleasant, but worth considering—that the source of low fertility rates is parental "egotism" induced by habits. From a broader perspective, the present analysis seems complementary to the related literature on Malthusian stagnation and demographic transition—e.g., Galor and Weil (2000) and Boldrin and Jones (2002). These contributions explicitly model the rise and overlap of different phases of development and characterize demographic transitions by considering the interactions between fertility, factor availability, and technology improvements. As we have emphasized the interactions between fertility and preferences, extending the present analysis to include technological transitions seems an interesting topic for future research.

The central role of bequests in our results suggests some similarities between our approach and that followed by Blackburn and Cipriani (2005), who argue that population dynamics are crucially governed by the direction of intergenerational transfers. Blackburn and Cipriani (2005) assume two-sided altruism in a model that generates development paths consistent with the Caldwell hypothesis [Caldwell (1982)]: low development is associated with young-to-old transfers and large family size, whereas high-development phases feature lower fertility and old-to-young transfers. With respect to Blackburn and Cipriani (2005), our analysis is different in both aims and means. At the formal level, the models differ in two important ways. First, we rule out two-sided altruism and focus on equilibrium paths with operative bequests—that is, transfers are "father-to-son" by construction. Second, habits are not considered in Blackburn and Cipriani (2005), whereas they represent the central feature of our analysis. At the conceptual level, our aim is different. Blackburn and Cipriani (2005) formalize the Caldwell hypothesis and reproduce the demographic transition along these lines by considering possible switch overs in the direction of transfers. In this paper, we asked whether persistently low fertility rates can also be explained by time-dependent preferences, arguing that the stage of development affects population growth because agents internalize previous standards of living into their bequest motives and fertility choices. Our reasoning is referred to the empirical evidence on the intertemporal behavior of consumers and is also consistent with less-recent historical facts. In the 18th century, the aristocracy pursued low-reproduction strategies in order to avoid the partition of the family property and maintain family status [Johansson (1987)]. In the 19th century, similar strategies were intentionally adopted by the bourgeoisie and landowners, gradually spreading across other social classes (Haines, 1992).

With respect to the literature on habit formation and economic growth, we can make four remarks. The first comment relates to the neoclassical model. In their seminal paper, Ryder and Heal (1973) show that habits do not modify the long-run

equilibrium of the economy, as the Ramsey model with infinite horizons predicts convergence toward the usual "modified golden rule." In Section 4, we have shown that habit formation modifies the long-run growth rate of the economy instead. The reason for our result is not the assumption of finite lifetimes,[12] but rather that of endogenous fertility: habits matter because they affect the equilibrium fertility rate and thereby the growth rate of aggregate output in any equilibrium with stationary income per capita. This result can be seen in parallel with that of Alvarez-Cuadrado et al. (2004), which asserts that habit formation becomes relevant for long-run growth when production possibilities are improved by exogenous productivity growth in the Ramsey model.

The second remark is related to the AK framework. Carrol et al. (1997) studied the role of habits in the standard AK model with exogenous fertility, showing that the degree of habit formation affects the long-run growth rate of the economy if the intertemporal elasticity of substitution differs from unity [Carrol et al. (1997), equations (17)–(19), p. 345]. The logarithmic case thus yields no growth effects of habits in the representative-agent framework—and this is also the case in our OLG model of Section 3, where the growth rate in aggregate variables, $\delta\mu A$, is independent of the degree of habit formation, ε. Specifying nonlogarithmic preferences in our AK model would reintroduce a missing link between habits and growth in *aggregate variables*. However, the aim of Section 3 is to emphasize the peculiar role of endogenous fertility and habit formation in determining fundamental changes in the growth rate of *per capita variables*. The assumption of logarithmic preferences allows us to study the various cases—i.e., extinction, overcrowding, and nondegenerate growth in Proposition 1—in isolation from transitional dynamics and elasticity-induced growth effects à la Carrol et al. (1997).

The third remark is related to models with overlapping generations. A comparable framework is that employed by Alonso-Carrera et al. (2007), who study a three-period OLG model with habit formation and inherited tastes. As we pointed out in Section 4.2, one of the results of Alonso-Carrera et al. (2007) can be identified with the first logical step of our reasoning: habits tend to contrast dynastic altruism. Differently from Alonso-Carrera et al. (2007)—who assume exogenous population growth—our model features endogenous fertility, which allows us to show that habits contrast population growth because habit formation prompts agents to reduce the cost of parental altruism in the second period of life.

The fourth remark refers to the links between habit formation and parental-based status. In related work, de la Croix and Michel (1999) analyze status-dependent preferences in order to study the effects of *inherited tastes*, assuming that the satisfaction index of a newborn agent is higher the higher is the gap between his current consumption and the level of "aspiration" determined by the consumption level of the *parents*. In the current model, this type of preferences would take the form $U(c_t, e_{t+1}, n_t, c_{t-1}) = \bar{u}(n_t) + u(c_t, c_{t-1}) + \beta v(e_{t+1})$, which clearly differs from (5). Indeed, the phenomenon of inherited tastes is different from—and it may be simultaneous to—that of habit formation, which refers to the

comparison that each agent makes between his own current and past consumption levels. It is possible to extend our model to include inherited tastes, though it is difficult to speculate on the consequences without explicit modeling: although aspirations would make the existence of positive bequests easier [Alonso-Carrera et al. (2007)], the interaction between aspirations and endogenous fertility choices is an open question that we leave for future research.

6. CONCLUSION

The recent growth literature proposes a number of explanations for (i) the low fertility rates currently observed in developed economies and (ii) the phenomenon known as "demographic transition." Theoretical contributions considered fertility choices under old-age security motives, or satisfaction-based approaches. In the standard framework, children provide direct utility to their parents, and the demographic transition may be due to increased productivity generated by technological progress, rising costs of child rearing, possible trade-offs between the quality and quantity of children, and the reversal in the direction of intergenerational transfers. In this paper, we have argued that economic development may affect population dynamics through a different channel, i.e., the interactions between fertility choices and habit formation in consumption. Most theoretical models with endogenous fertility assume that consumption choices are based on standard time-separable preferences, though a growing body of empirical evidence suggests that preferences are status dependent in reality. Building on this point, we have studied fertility choices and habit formation in an overlapping generations model, assuming that the opportunity cost of having children is determined by bequests. In this setting, habits contrast dynastic altruism through status-related effects. Our reasoning may be linked to the empirical observation that low-reproduction strategies have historically been intentional choices aimed at preserving the family property and status [Johansson (1987); Haines (1992)].

A first general result is that habits reduce population growth. The reason is that, due to habit formation, consumption choices are biased in favor of second-period consumption, and agents aim at reducing the loss in second-period utility induced by bequests. Being able to choose the number of children, individuals reduce fertility in order to make parental altruism less costly. We have studied this mechanism under two alternative technology specifications. In the neoclassical model, habits modify the economy's growth rate and generate transitional dynamics in fertility that are compatible with the phenomenon of demographic transition. In the long run, stationary income per capita is associated with either increasing or decreasing population and output, depending on the strength of habits. In the AK specification, growing population and increasing consumption per capita require that the habit coefficient lie within definite boundaries; outside the critical interval, positive growth is associated with either declining consumption due to overcrowding or extinction paths with declining population. In both frameworks, habits are responsible for the fertility decline, suggesting that status-dependent preferences

may have played an important role in the demographic transition observed in developed economies.

NOTES

1. Spain and Italy are nowadays among the countries with the lowest fertility rates in the world. The total fertility rate fell from around 3 in 1960 to 1.2 in 2000, which is well below the replacement level of 2.1 [Kohler, Billari, and Ortega (2002)].

2. For example, the idea that fertility declined in response to a reduction in infant mortality may be consistent with observed dynamics in Germany and Sweden. However, in the United States and France, reduced mortality followed fertility decline [Galor (2005)], suggesting that the contribution of observed mortality rates to the demographic transition is limited in these cases [Mateos-Planas (2002)].

3. The operativeness of bequests is generally linked to technological and preference parameters according to the following logic. Because making a positive transfer to successors implies reduced capital accumulation, agents find it optimal only if the resulting increase in utility through the altruistic term more than compensates for the loss in second-period capital income. Hence, bequests are operative in the equilibrium if the degree of altruism (denoted by δ in the present paper) is sufficiently high relative to the technological parameters determining capital profitability: see, e.g., the condition derived in de la Croix and Michel (2002, p. 253).

4. Thibault (2000) shows that, under specifications (2)–(3), the equilibrium with operative bequests: (i) exists only if the conditions for dynamic efficiency are satisfied; and (ii) is unique. Moreover, Thibault (2000) shows that (iii) the equilibrium with zero bequests coincides with the Diamond (1965) equilibrium—so it can be dynamically inefficient. de la Croix and Michel (2002, Section 5.1.4) discuss the conditions under which the equilibrium with operative bequests coincides with the modified golden rule à la Ramsey–Cass–Koopmans and implies policy neutrality (Ricardian equivalence). In our analysis of the neoclassical model, we specify numerical parameters that allow the economy to be dynamically efficient in the long run: see Section 4.2.

5. In this specification, the utility of each agent is affected by the utility of each child with a fixed degree of altruism, δ. As the budget constraint is expressed in terms of per-child bequest, the interpretation is that individual choices regarding the amount of *per-child* gifts, b_{t+1}, are determined on the basis of *per-child* utility, W_{t+1}. An equivalent interpretation is that parents care about the average utility of each child, as explained in Doepke and Zilibotti (2005).

6. We choose this strategy because the aim of the present analysis is to study not the operativeness of bequest motives but rather the interactions between habit formation and fertility rates in situations in which bequests are operative. Operativeness is studied in detail in de la Croix and Michel (1999, 2002) and Alonso-Carrera et al. (2007) in related models with exogenous population growth.

7. In related work, de la Croix and Michel (1999) analyze status-dependent preferences in order to study the effects of *inherited tastes*, which is a different phenomenon with respect to habit formation and reflects into different specifications of first-period utility. See Section 5 on this point.

8. Result (29) is in line with the findings of Alonso-Carrera et al. (2007, Section 3), who show that habits make the operativeness of bequests more difficult and there is a critical level of the degree of habit formation above which agents become selfish.

9. See Figure 1. For a baseline nondegenerate value of $\varepsilon = 4\%$, gross per-period rates of growth of aggregate output, population, and consumption per capita, respectively, equal $Y_{t+1}/Y_t = 1.2$, $n_{ss} = 1.11$, and $c_{t+1}/c_t = 1.085$. Setting $\varepsilon = 6\%$ yields an extinction path with declining population and excessive consumption growth ($n_{ss} = 0.88$ and $c_{t+1}/c_t = 1.36$). The mirror case is obtained with $\varepsilon = 2\%$, associated with overcrowding ($n_{ss} = 1.36$ and $c_{t+1}/c_t = 0.88$).

10. The existence of habits ($\varepsilon > 0$) makes the dynamics of capital per worker dependent on the expected interest rate, as the presence of $f'(k_{t+1})$ on the right-hand side of (50) elucidates. Hence, k_{t+1} is (only) implicitly given by (50). In the absence of habits ($\varepsilon = 0$), equation (50) reduces to the

familiar resource constraint

$$k_{t+1} = \frac{1}{n_t}\left[f(k_t) - \frac{1}{\lambda_t}\left(1+\frac{\beta}{\delta}\right)\right],$$

where k_{t+1} does not appear anymore in the right-hand side.

11. As habits modify the consumption *propensity*, the reallocation effect of habit formation reverberates in each period in which agents save. The consequences for consumption *levels* become evident only after this "cumulative effect" on the capital stock has become quantitatively relevant. This is why the effects of habits on consumption levels seem relatively small in the short run: in the first two periods reported in Figure 3, the growth process is in its early stages.

12. If we drop the assumption of endogenous fertility, we obtain an OLG model with dynastic altruism that yields predictions identical to the Ramsey model with infinite horizons, provided that bequests are operative in each period along the equilibrium path [see de la Croix and Michel (2002, Chapter 5)].

13. Explosive trajectories can ruled out following the standard argument. If $x_0 < x_{ss}$, capital will become negative in finite time, violating the aggregate constraint of the economy. If $x_0 > x_{ss}$, capital grows faster than consumption of the young, i.e., $\lim_{t\to\infty}(K_{t+1}/K_t) > \delta\mu A > \delta$, but this path would violate the transversality condition $\lim_{t\to\infty}\delta^t K_t \leq 0$.

14. As shown in expression (47), when $\varepsilon > 0$, the curve $g^b(n_*)$ is bounded above by $\lim_{\varepsilon\to 0} g^b(n_*)$. To have an intersection $g_a = g_b$, it is thus necessary to have $\lim_{\varepsilon\to 0} g^b(n_*) > 0$, which in turn requires satisfying (A.18).

15. Setting $g^b(0) > 0$ and substituting the last term in (A.20), we obtain $\delta > \frac{1-\mu}{1-\mu\delta}$, which can be rewritten as $\mu(1-\delta^2) > 1-\delta$. Substituting $1-\delta^2 = (1-\delta)(1+\delta)$ and rearranging terms, we obtain expression (A.21) and therefore expression (A.22).

16. Notice that $\varepsilon < 1$ is a sufficient condition although it is not strictly necessary for the above argument. In general, $\text{AP}_b > \text{AP}_a$ is obtained whenever $\varepsilon < \frac{1+\sigma}{1-\sigma}$. We emphasize the condition $\varepsilon < 1$ as this is a reasonable assumption.

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APPENDIX

Proof of Lemma 1. From (19), condition (17) can be written as

$$e_{t+1}/c_t = \mu A\beta + \varepsilon\,(1+\beta)\,. \tag{A.1}$$

Plugging $v_{e_{t+1}} = (e_{t+1} - \varepsilon c_t)^{-1}$ into (10) and using (A.1) to substitute e_{t+1} yields

$$c_t\lambda_t = \mu A\,(\mu A + \varepsilon)^{-1}. \tag{A.2}$$

Combining (A.2) with (11) we obtain

$$c_{t+1}n_t = c_t\delta\mu A. \tag{A.3}$$

Because $C_{t+1}/C_t = n_t\,(c_{t+1}/c_t)$, the above expression implies a constant growth rate of aggregate consumption of the young, $\delta\mu A$. Moreover, substituting (A.3) in (A.1), and using $N^y_{t+1} = n_t N^o_t$, we obtain (23), which completes the proof. ■

Proof of Lemma 2. Define $x_t \equiv K_t/C_t$. Dividing both sides of (21) by C_{t+1}, and using (22)–(23) to eliminate E_t, we obtain

$$x_{t+1} = \frac{1}{\delta\mu}x_t - \frac{\mu A\,(\beta+\delta) + \varepsilon\,(1+\beta)}{(\delta\mu A)^2}, \tag{A.4}$$

which displays a unique steady-state solution

$$x_{ss} = \frac{\mu A\,(\beta+\delta) + \varepsilon\,(1+\beta)}{(1-\delta\mu)\,\delta\mu A^2}. \tag{A.5}$$

For a given K_0, the initial condition x_0 is determined by the jump-variable C_0. Because $\delta\mu < 1$, the steady state is unstable and all the trajectories starting from $x_0 \neq x_{ss}$ are explosive and nonoptimal.[13] Hence, variable x jumps at the steady-state level x_{ss} at time zero and is constant thereafter. From Lemma 1 and constraint (21), $x_t = x_{ss}$ at each t implies balanced growth from time zero onward in aggregate variables, with K_t, Y_t, C_t, and E_t all growing at the constant rate $\delta\mu A$. Because $K_t/C_t = x_{ss}$, we have $Y_t/C_t = (AK_t/C_t)^{-1} = (Ax_{ss})^{-1}$, which implies (24) after substituting (A.5). ■

Derivation of (26). Substituting $r_{t+1} = \mu A$ and (17) in (6) yields

$$b_{t+1} = \frac{\mu A}{n_t}\left[w_t + b_t - c_t\frac{(\mu A+\varepsilon)\,(1+\beta)}{\mu A}\right].$$

Dividing both sides by c_{t+1} and using (A.3), we have

$$z_{t+1} = \frac{1}{\delta}\left[\frac{w_t}{c_t} + z_t - \frac{(\mu A+\varepsilon)\,(1+\beta)}{\mu A}\right]. \tag{A.6}$$

Because $w_t/c_t = (1-\mu)\,(Y_t/C_t)$, the first term in brackets equals $(1-\mu)\,\psi^{-1}$, implying equation (26) in the text.

Proof of Proposition 1. From (16), plugging $\bar{u}_{n_t} = \gamma\sigma n_t^{\sigma-1}$ into the first-order condition (13) gives $\lambda_t b_{t+1} = \gamma\sigma\mu A n_t^{\sigma-1}$. Substituting expressions (A.1) and (A.3), we obtain

$$\frac{b_{t+1}}{c_{t+1}} = \frac{\gamma\sigma\,(\mu A+\varepsilon)}{\delta\mu A} n_t^{\sigma}. \tag{A.7}$$

Because $b_{t+1} = z_{ss}c_{t+1}$ in each period, expression (A.7) implies (30). Because $\partial z_{ss}/\partial\varepsilon < 0$, it follows from (30) that $\partial n_{ss}/\partial\varepsilon < 0$. ■

Proof of Proposition 2. Because the growth rate of aggregate output $Y_{t+1}/Y_t = \delta\mu A$ is independent of habit formation, the nonambiguous sign of the derivative $\partial n_{ss}/\partial\varepsilon < 0$ implies that there generally exist a couple of critical levels ε' and ε'', with $\varepsilon' < \varepsilon''$, such that $\varepsilon = \varepsilon'$ implies $n_{ss} = \delta\mu A > 1$, and $\varepsilon = \varepsilon''$ implies $n_{ss} = 1$. For reasonable parameter values, both ε' and ε'' are strictly positive, implying cases (i), (ii), and (iii)—see Figure 1. ■

Derivation of (36)–(37). Equation (37) is obtained as follows. Substituting $v_{e_{t+1}} = (e_{t+1} - \varepsilon c_t)^{-1}$ in (10) we obtain $\beta n_t = \delta\lambda_{t+1}(e_{t+1} - \varepsilon c_t)$. Substituting $\delta\lambda_{t+1} = \lambda_t n_t r_{t+1}^{-1}$ from (11) we thus have $\beta r_{t+1} = \lambda_t(e_{t+1} - \varepsilon c_t)$. Substituting $e_{t+1} = c_t[\beta r_{t+1} + \varepsilon(1+\beta)]$ from (17), we obtain

$$\frac{\beta}{c_t\,[\beta r_{t+1} + \varepsilon\,(1+\beta)] - \varepsilon c_t} = \frac{\lambda_t}{r_{t+1}},$$

which reduces to (37) after rearranging terms and substituting $r_{t+1} = f'(k_{t+1})$. Equation (36) is obtained as follows. From (34) it follows that

$$c_t + \frac{e_{t+1}}{r_{t+1}} = c_t + \frac{c_t\,[\beta r_{t+1} + \varepsilon\,(1+\beta)]}{r_{t+1}} = c_t\,(1+\beta)\left(\frac{r_{t+1}+\varepsilon}{r_{t+1}}\right). \tag{A.8}$$

Substituting (A.8) in (6), we obtain

$$b_{t+1} = \frac{r_{t+1}}{n_t}\,(w_t + b_t) - c_t\,(1+\beta)\left(\frac{r_{t+1}+\varepsilon}{n_t}\right). \tag{A.9}$$

Now rewrite condition (35) as

$$b_{t+1} = \frac{u_{n_t}}{\lambda_t} r_{t+1} \text{ and } b_t = \frac{u_{n_{t-1}}}{\lambda_{t-1}} r_t, \tag{A.10}$$

and combine (A.10) with (A.9) to get

$$u_{n_t} = \lambda_t \frac{1}{n_t}\left(w_t + \frac{u_{n_{t-1}}}{\lambda_{t-1}} r_t\right) - \lambda_t c_t\,(1+\beta)\,\frac{1}{n_t}\left(\frac{r_{t+1}+\varepsilon}{r_{t+1}}\right). \tag{A.11}$$

Substituting (37) in (A.11) yields

$$u_{n_t} n_t = \lambda_t w_t + u_{n_{t-1}} \frac{\lambda_t}{\lambda_{t-1}} r_t - (1+\beta), \tag{A.12}$$

and, substituting $\frac{\lambda_t}{\lambda_{t-1}} r_t = \frac{n_{t-1}}{\delta}$, we obtain

$$u_{n_t} n_t = u_{n_{t-1}} n_{t-1} \frac{1}{\delta} + \lambda_t w_t - (1+\beta).$$

Substituting $w_t = (1-\mu)\,f(k_t)$ and $u_{n_t} n_t = \sigma\gamma n_t^{\sigma}$ we obtain expression (36).

Derivation of (44)–(45). Using (39) to substitute e_* in (38), and using (42) to substitute c_* in the resulting expression, we have

$$n_* k_* = f(k_*) - \left(\frac{1}{\lambda_*}\right) \frac{f'(k_*)}{f'(k_*) + \varepsilon} \left[\frac{n_* + \beta f'(k_*) + \varepsilon(1+\beta)}{n_*}\right]. \tag{A.13}$$

Because $f'(k_*) = \mu H k_*^{\mu-1}$, condition (40) implies $f'(k_*) = n_*/\delta$ and $k_* = (\mu H \delta / n_*)^{\frac{1}{1-\mu}}$. Substituting these results in (A.13) and solving for λ, we obtain

$$\lambda_* = \frac{n_* + n_*(\beta/\delta) + \varepsilon(1+\beta)}{n_* + \varepsilon\delta} \left[H\left(\frac{\mu H \delta}{n_*}\right)^{\frac{\mu}{1-\mu}} - n_*\left(\frac{\mu H \delta}{n_*}\right)^{\frac{1}{1-\mu}}\right]^{-1}. \tag{A.14}$$

From (41) we have

$$n_*^{\sigma} = \left(\frac{\delta}{1-\delta}\right) \frac{1}{\sigma\gamma} \left[(1+\beta) - (1-\mu) f(k_*) \lambda_*\right],$$

where we can substitute (A.14) and $f(k_*) = H(\mu H \delta / n_*)^{\frac{\mu}{1-\mu}}$ to obtain

$$n_*^{\sigma} = \frac{\delta(1+\beta)}{\sigma\gamma(1-\delta)} - \frac{(1-\mu)}{\sigma\gamma(1-\delta)(1-\mu\delta)} \left[\frac{n_*(\delta+\beta) + \varepsilon\delta(1+\beta)}{n_* + \delta\varepsilon}\right]. \tag{A.15}$$

Defining $g^a(n_*) = n_*^{\sigma}$ and $g^b(n_*)$ as the right-hand side of (A.15), this equilibrium condition can be rewritten as $g^a(n_*) = g^b(n_*)$.

Proof of Proposition 3. Expression (45) implies

$$\lim_{\varepsilon \to 0} g^b(n_*) = \frac{\delta(1+\beta)}{\sigma\gamma(1-\delta)} - \frac{(1-\mu)(\delta+\beta)}{\sigma\gamma(1-\delta)(1-\mu\delta)}, \tag{A.16}$$

As $\varepsilon \to 0$, $g^b(n_*)$ becomes independent of n_* and reduces to a horizontal line in the $g^i - n_*$ plane. Because $0 < \sigma < 1$, the function $g^a(n_*) = n_*^{\sigma}$ is strictly increasing in the $g^i - n_*$ plane and satisfies $g^a(0) = 0$. This implies that only one value $n_* > 0$ may satisfy the equilibrium condition $g^a = g^b$ (see Figure 2). Imposing (43) we obtain

$$n_* = \left\{\frac{1}{\sigma\gamma(1-\delta)} \left[\delta(1+\beta) - \frac{(1-\mu)(\delta+\beta)}{(1-\mu\delta)}\right]\right\}^{1/\sigma}. \tag{A.17}$$

The term in square brackets implies that there exists a unique well-defined equilibrium when parameters satisfy

$$\frac{\delta(1+\beta)}{\delta+\beta} > \frac{1-\mu}{1-\mu\delta}. \tag{A.18}$$

When (A.18) is violated, there is no well-defined equilibrium, because the right-hand side of (A.16) is negative. The unique equilibrium is feasible if and only if the right-hand side of (A.17) is greater than δ. ■

Proof of Proposition 4. Notice that, when $\varepsilon > 0$, the existence of an equilibrium is linked to the case of inactive habits: in order to have an equilibrium it is necessary that (A.18) is satisfied.[14] As regards the characteristics of candidate equilibria, the reasoning is

as follows. As shown in the main text, both $g^a(n_*)$ and $g^b(n_*)$ are strictly increasing and concave in n_* for any $\varepsilon > 0$. The vertical intercepts are

$$g^a(0) = \lim_{n_* \to 0} g^a(n_*) = 0, \tag{A.19}$$

$$g^b(0) = \lim_{n_* \to 0} g^b(n_*) = \frac{\delta(1+\beta)}{\sigma\gamma(1-\delta)} - \frac{(1-\mu)(1+\beta)}{\sigma\gamma(1-\delta)(1-\mu\delta)}. \tag{A.20}$$

Expression (A.20) implies that[15]

$$\mu > (1+\delta)^{-1} \Longrightarrow g^b(0) > g^a(0) = 0, \tag{A.21}$$

$$\mu < (1+\delta)^{-1} \Longrightarrow g^b(0) < g^a(0) = 0. \tag{A.22}$$

Case (A.21) is characterized by the fact that the vertical intercept of g^b is higher than $g^a(0) = 0$. Case (A.22) is characterized by the fact that the vertical intercept of g^b is lower than $g^a(0) = 0$. The issue of the number of intersections can be addressed by comparing the curvature of the two functions as captured by the Arrow–Pratt coefficients (AP_a and AP_b), respectively, equal to

$$\mathrm{AP}_a = -\frac{g^a(n_*)'' n_*}{g^a(n_*)'} = 1 - \sigma, \tag{A.23}$$

$$\mathrm{AP}_b = -\frac{g^b(n_*)'' n_*}{g^b(n_*)'} = \frac{2n_*}{n_* + \delta\varepsilon}. \tag{A.24}$$

While AP_a is constant, positive, and smaller than unity, expression (A.24) implies

$$\frac{\partial \mathrm{AP}_b}{\partial n_*} = \frac{2\delta\varepsilon}{(n_* + \delta\varepsilon)^2} > 0, \tag{A.25}$$

$$\lim_{n_* \to 0} \mathrm{AP}_b = 0 < \mathrm{AP}_a, \tag{A.26}$$

$$\lim_{n_* \to \infty} \mathrm{AP}_b = 2 > \mathrm{AP}_a, \tag{A.27}$$

Expressions (A.25)–(A.27) imply that the curvature of $g^b(n_*)$ falls short of that of $g^a(n_*)$ in the origin and then monotonically increases, eventually exceeding that of $g^a(n_*)$ for higher values of n_*. This implies the following results:

(a) Suppose that $\mu > (1+\delta)^{-1}$. From (A.21) we have $g^b(0) > g^a(0)$. Given this, expressions (A.25)–(A.27) imply that the condition $g^a = g^b$ can be satisfied only by a unique intersection where $g^b(n_*)$ cuts $g^a(n_*)$ from above, as shown in Figure 2 (a). This intersection characterizes a feasible equilibrium provided that parameters imply $n_* > \delta$.

(b) Suppose that $\mu < (1+\delta)^{-1}$. From (A.21) we have $g^b(0) < g^a(0)$, and expressions (A.25)–(A.27) imply that the condition $g^a = g^b$ can be satisfied by two intersections, $n' > 0$ and $n'' > 0$, with $n' < n''$. As shown in Figure 2(b), the "high" intersection n'' has the same properties as the unique equilibrium arising in diagram (a): $g^a = g^b$ is satisfied with $g^b(n_*)$ cutting $g^a(n_*)$ from above; the "low" intersection n', instead, satisfies the condition $g^a = g^b$ with $g^b(n_*)$ cutting $g^a(n_*)$ from below.

Given cases (a) and (b), if we can rule out n' as unfeasible, we obtain the proof that the steady-state equilibrium of the economy is unique and is characterized by an intersection in which $g^b(n_*)$ cuts $g^a(n_*)$ from above. The proof is as follows. Suppose that parameters are such that $g^a = g^b$ is satisfied with $n_* = \delta$. From (A.24), this intersection would be characterized by

$$n_* = \delta \Longrightarrow \mathrm{AP}_b = \frac{2}{1+\varepsilon}. \tag{A.28}$$

From (A.23) and (A.28), it follows that, for any $\varepsilon < 1$, we have $\mathrm{AP}_b > \mathrm{AP}_a$.[16] That is, an intersection $g^a = g^b$ with $n_* = \delta$ can only be satisfied with $g^b(n_*)$ cutting $g^a(n_*)$ from above. This reasoning can be repeated for any intersection in which $n_* > \delta$: because AP_b is increasing in n_* while AP_a is fixed, any intersection $g^a = g^b$ with $n_* > \delta$ is characterized by $g^b(n_*)$ cutting $g^a(n_*)$ from above. It follows from this result that the candidate equilibrium n' that arises in case (b) with $g^b(n_*)$ cutting $g^a(n_*)$ from below is necessarily characterized by $n' < \delta$, and this is not a feasible equilibrium. The conclusion is that, irrespective of whether we are in case (a) or in case (b), there may exist only one feasible steady state with $n_* > \delta$, and this equilibrium is represented by an intersection in which $g^b(n_*)$ cuts $g^a(n_*)$ from above. ■

Proof of Lemma 3. If $n_{\max} > 1$, there exists a critical value of habit formation $\varepsilon_\delta > 0$ for which we obtain an equilibrium $n_* = \delta$. Being $g^b(n_*; \varepsilon)$ strictly declining in ε, well-defined equilibria are characterized by values of ε lying in the interval $0 \leq \varepsilon < \varepsilon_\delta$. Because $\delta < 1$, the equilibrium $n_* = 1$ is feasible, i.e., it lies in the interior of the relevant region; see Figure 2. Because $\partial g^b / \partial \varepsilon < 0$, the equilibrium $n_* = 1$ is associated with a critical value of the habit coefficient $\bar{\varepsilon}$ such that $0 < \bar{\varepsilon} < \varepsilon_\delta$. As a consequence, there exists a subset of well-defined equilibria associated with growing population (characterized by $0 < \varepsilon < \bar{\varepsilon}$ and $n_* > 1$) and another subset associated with declining population (characterized by $\bar{\varepsilon} < \varepsilon < \varepsilon_\delta$ and $n_* < 1$). ■

Proof of Proposition 5. From Lemma 3, if ε falls in the intervals mentioned in cases (i), (ii), and (iii), we respectively have increasing, decreasing, and constant population. The proof is completed by recalling that $Y_{t+1}/Y_t = n_*$ holds in the steady-state by virtue of (49). ■

Derivation of (50)–(52). Equations (51)–(52) directly follow from (35)–(36). Equation (50) can be obtained as follows. Rewriting (52) at time t and substituting $\lambda_t = r_{t+1}[c_t(r_{t+1} + \varepsilon)]^{-1}$ from (37), we get

$$\lambda_{t-1} c_t = \frac{\delta r_{t+1} r_t}{n_{t-1}\,(r_{t+1} + \varepsilon)}. \tag{A.29}$$

Substituting $\lambda_{t-1} = r_t[c_{t-1}(r_t + \varepsilon)]^{-1}$ from (37) and $e_t = c_{t-1}[\beta r_t + \varepsilon(1+\beta)]$ from (17), we have

$$\frac{e_t}{n_{t-1}} = c_t \frac{(r_{t+1} + \varepsilon)\,[\beta r_t + \varepsilon\,(1+\beta)]}{\delta r_{t+1}\,(r_t + \varepsilon)}. \tag{A.30}$$

Plugging (A.30) in (33), and substituting (37) to eliminate c_t, we obtain

$$n_t k_{t+1} = f\,(k_t) - \frac{f'\,(k_{t+1})}{\lambda_t\,(f'\,(k_{t+1}) + \varepsilon)} \left\{ 1 + \frac{(f'\,(k_{t+1}) + \varepsilon)\,[\beta f'\,(k_t) + \varepsilon\,(1+\beta)]}{\delta f'\,(k_{t+1})\,(f'\,(k_t) + \varepsilon)} \right\}, \tag{A.31}$$

which implies (50) in the text.

Simulation Method. Backward iteration requires us to determine an initial value to evaluate the time-reversed system up to a termination criterion and to revert the obtained sequence of solutions. Because the analytical selection of an initial value on the stable manifold is generally not available, the initial value is given by a point in the neighborhood of the steady state. A good approximation is obtained by making use of the stable eigenvector of the Jacobian J_* which is tangent to the saddle path at the stationary solution (k_*, λ_*, n_*). We thus set

$$\begin{vmatrix} k_0 \\ \lambda_0 \\ n_0 \end{vmatrix} = \begin{vmatrix} k_* \\ \lambda_* \\ n_* \end{vmatrix} + d^i d^d,$$

where d^i represents the initial deviation from the equilibrium and d^d defines the direction. The initial values (k_0, λ_0, n_0) are set equal to $(k_{t+1}, \lambda_{t+1}, n_{t+1})$, and the system (50)–(52) is solved backward for (k_t, λ_t, n_t), and so on. In the simulation, the approximations of the linearized system proved to be a good first guess for the solution of the system. The linearized system is

$$\begin{vmatrix} k_t \\ \lambda_t \\ n_t \end{vmatrix} = P \begin{vmatrix} A_1 & \nu_1^t \\ A_2 & \nu_2^t \\ A_3 & \nu_3^t \end{vmatrix} + \begin{vmatrix} k_* \\ \lambda_* \\ n_* \end{vmatrix},$$

where P contains the eigenvectors p_1, p_2, p_3 and A_1, A_2, A_3 represent arbitrary constants. Because $\nu_1, \nu_2 > 1$, it follows immediately that $A_1 = A_2 = 0$, such that

$$\begin{aligned} k_t &= p_{3,1} A_3 \nu_3^t + k_*, \\ \lambda_t &= p_{3,2} A_3 \nu_3^t + \lambda_*, \\ n_t &= p_{3,3} A_3 \nu_3^t + n_*. \end{aligned}$$

Because k_0 is given, A_3 is obtained as $\frac{k_0 - k_*}{e_{3,1}} = A_3$.

Proof of Proposition 6. Multiplying both terms of (50) by λ_{t+1} and substituting (51), we obtain

$$\delta \lambda_{t+1} k_{t+1} f'(k_{t+1}) = \lambda_t f(k_t) - \frac{f'(k_{t+1})}{f'(k_{t+1}) + \varepsilon} - \frac{\beta f'(k_{t+1}) + \varepsilon(1+\beta)}{\delta(f'(k_{t+1}) + \varepsilon)}. \tag{A.32}$$

Because $f'(k_{t+1}) = r_{t+1}$ and $k_{t+1} f'(k_{t+1}) = \mu f(k_{t+1})$, we can define the shadow value of output per capita as $Q_t \equiv \lambda_t y_t = \lambda_t f(k_t)$ and rewrite (A.32) as

$$Q_{t+1} = \frac{1}{\mu\delta}\left[Q_t - \frac{r_{t+1}}{r_{t+1} + \varepsilon} - \frac{\beta r_{t+1} + \varepsilon(1+\beta)}{\delta(r_{t+1} + \varepsilon)} \right]. \tag{A.33}$$

If $\varepsilon = 0$, this expression reduces to

$$Q_{t+1} = \frac{1}{\mu\delta}\left(Q_t - \frac{\beta + \delta}{\delta} \right), \tag{A.34}$$

which displays a unique steady-state $Q_{ss} = \frac{\beta+\delta}{\delta(1-\mu\delta)}$. Because $\mu\delta < 1$, this steady state is unstable, and explosive paths can be ruled out as suboptimal: if $\lim_{t\to\infty} Q_t = -\infty$, capital per worker must become negative in finite time, whereas $\lim_{t\to\infty} Q_t = +\infty$ would

violate the transversality condition $\lim_{t\to\infty} \lambda_t k_t = 0$. Hence, when habits are inactive, the shadow value of output per capita jumps at Q_{ss} at time zero and stays there forever [this result is intuitive, because (37) implies that, when $\varepsilon = 0$, the shadow value of first-period consumption is constant and equal to unity at each point in time]. The fact that $Q_t = Q_{ss}$ in each t implies that fertility rates exhibit no transition. To see this, rewrite (36) as

$$n^{\sigma}_{t+1} = \frac{1}{\delta} n^{\sigma}_{t} + \frac{1}{\sigma\gamma}\left[(1-\mu)\, Q_{t+1} - (1+\beta)\right]. \tag{A.35}$$

If $\varepsilon = 0$ we have $Q_{t+1} = Q_{ss}$, and (A.35) displays an unstable steady state in fertility rates. Ruling out explosive paths that would make n_t diverge to plus/minus infinity, the only possible equilibrium with inactive habits is characterized by $Q_t = Q_{ss}$ and $n_t = n_{\max}$ at each point in time, which completes the proof. ■

Derivation of (53). From (35) and (37)

$$\frac{c_{t+1}}{c_t} = \frac{\delta\,(r_{t+1}+\varepsilon)\, r_{t+2}}{n_t\,(r_{t+2}+\varepsilon)}.$$

Using (34) to substitute c_t, and rearranging terms, we have

$$\frac{c_{t+1}}{e_{t+1}} = \frac{\delta\,(r_{t+1}+\varepsilon)\, r_{t+2}}{n_t\,(r_{t+2}+\varepsilon)\,[\beta r_{t+1} + \varepsilon\,(1+\beta)]}.$$

Multiplying both sides by N^{y}_{t+1}/N^{o}_{t+1}, and recalling that $N^{o}_{t+1} = N^{y}_{t}$, we have

$$\frac{C_{t+1}}{E_{t+1}} = \frac{\delta\,(r_{t+1}+\varepsilon)\, r_{t+2}}{(r_{t+2}+\varepsilon)\,[\beta r_{t+1} + \varepsilon\,(1+\beta)]},$$

which implies (53) in the text.